MATH 396 FINAL EXAM

Note: Work singly. Please only set up the solutions to the problems. E.g. $P(X\text{happens}) = \frac{1}{\sqrt{2\pi}} \int_2^7 e^{-t^2/2} dt$ is an answer preferable to the decimal approximation, as are expressions using symbols like $\chi^2_{1-\alpha/2,n-1}, t_{\alpha/2,n-1} \cdots$

1) Recall that in the derivation of the F-distribution's *pdf* we computed the *cdf* $F_{U/V}(t) = P(U \leq tV) = \int\int_R f_U f_V$ with $R = \{(u,v) \mid u \leq tv\} \subset \mathbb{R}^2_+$ (assuming the independence of the random variables U and V). Find the *pdf* for the random variable UV where U and V are independent random variables with a uniform *pdf* $f_U(t) = f_V(t) = 1/\theta$.

2) For the previous problem, compute $E(UV)$ and $Var(UV)$.

3) Use the method of moments and the method of maximum likelihood to estimate the parameter θ characterizing the *pdf* from the first problem. Determine whether either of these estimators is biased and if so, modify it so that it is not. The following integrals may be useful: $\int_0^\alpha t^n \ln(\alpha/t) dt = \frac{\alpha^{n+1}}{(n+1)^2}$.

4) For the results of problem 3, determine which of the estimators for θ is the more efficient. *Bonus*: Use Cramér-Rao to evaluate how efficient each of the estimators above is in absolute terms.

5) Let $\{1,2,3,4,5,4,3,2,1\}$ be a random sample from a normal distribution. Construct a 95% confidence interval for the variance of the model. Use the annoying notation $\chi^2_{1-\alpha/2,n-1} \cdots$, not the actual numbers from tables.

6) *Cf. Theorem 7.5.1 from Larsen and Marx.* For the data from the previous problem, construct a 95% confidence interval for the mean.