NUMERICAL ANALYSIS EXAM I

1) With a diagram, if necessary, describe Newton's method.

2) Give a full description of Simpson's rule. I.e. Find the area under a parabola through three given points in terms of the points, *etc.*

3) Generally speaking, what is the difference between closed and open integration methods? Which and why is one preferred over the other for solving improper integrals? Give a way to evaluate

$$\int_0^\infty \frac{dx}{1+x^4}.$$

4) Describe Euler's method for a scalar, first-order ordinary differential equation.

5) Compute

$$\exp\left[t\begin{pmatrix} 0 & 1 \\ 5 & 2 \end{pmatrix}\right]\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

6) To which second-order *IVP* is problem 5 equivalent?

7) How would you tell a computer to differentiate a function?

8) Compute a good approximation to the time-ordered exponential

$$\mathcal{T}_0^1 \exp\left[\begin{pmatrix} 0 & 1 \\ 5t & 2\sin t \end{pmatrix}\right]\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and relate it to a scalar *IVP*. Next, solve the *IVP* by series development (as in your differential equations class) and compare the two answers, say, by examining their values at $t = 1$. Which is less effort to get a good solution? Compare both your solutions to what *Maple* gets numerically.