In-Class Exercise I

November 6, 2003

Let d be a metric on a set X. As you know, this means that for any $x, y, z \in X$ we have

- $d(x, y) \geq 0$
- $d(x, y) = 0 \Rightarrow x = y$
- $d(x, y) = d(y, x)$
- $d(x, z) \leq d(x, y) + d(y, z)$.

Now let $f : \mathbb{R} \to [0, \infty)$ be a function satisfying the following three properties (if f is twice-differentiable, this means that $f' > 0$ and $f'' < 0$).

- $f(0) = 0$
- $x < y \Rightarrow f(x) < f(y) \quad$ (f is strictly increasing)
- $f(x + y) \leq f(x) + f(y) \quad$ (f is subadditive).

Your job is to conclude that $D(x, y) = f(d(x, y))$ is a metric on X. I.e. show in the space provided that the following properties of D hold for all $x, y, z \in X$:

- $f(d(x, y)) \geq 0$
- $f(d(x, y)) = 0 \Rightarrow x = y$
- $f(d(x, y)) = f(d(y, x))$
- $f(d(x, z)) \leq f(d(x, y)) + f(d(y, z))$.