ANALYSIS I HOMEWORK I

In Rudin's *Principles of Mathematical Analysis*, p 21, do exercises 2, 4, 8, 9, 11-15, 17, 18.

In addition, solve the following:

1) Prove or disprove: r, s irrational $\Rightarrow$ $r + s$ irrational.

2) Show that $\mathbb{Z}_5$ is a field. $\mathbb{Z}_n = \{0, 1, \ldots,\ n-1\}$ with addition and multiplication defined modulo n. Is it possible to order $\mathbb{Z}_5$ so that it becomes an ordered field?

3) Here I propose a solution for problem 1, p 21 from Rudin:
Claim: $0 \neq r \in \mathbb{Q}$ and $x \notin \mathbb{Q}\ \Rightarrow rx \notin \mathbb{Q}$
Proof: $rx = p/q, r = s/t,\ \exists p, q, s, t \in \mathbb{Z}\ \Rightarrow x = rx/r = pt/qs \in \mathbb{Q}$.

a) Is this a correct proof?
b) Even if it is not a correct proof you should be able to see whether it was an attempt at a forward proof, contrapositive, contradiction, or counterexample. Which, then?
c) If the proof is incorrect, correct it. If it is OK, explain all the implications and the logical structure (i.e. A and B $\Rightarrow$ C proven by...).
d) Get the result for $r + x$.