REAL ANALYSIS I FINAL EXAM

1) Show that if p is a limit point of a Cauchy sequence then it is unique.

2) Show that $\sum_k a_k < \infty \Rightarrow \sum_k a_k^2 < \infty$ but *not* conversely.

3) Assuming Taylor's theorem, prove the binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) a^{n-k} b^k. \tag{1}$$

Hint: Expand $f(x) = (1+x)^n$.

4) Determine $\lim_{n\to\infty}(1+\frac{2}{n})^n$ and $\lim_{n\to\infty}(1-\frac{1}{n})^n$.

5) Let f_k be the kth term in Fibonacci's sequence $f_0 = f_1 = 1,\ f_k = f_{f-1} + f_{k-2}$.
Show that

$$\lim_{n\to\infty} \frac{f_n}{f_{n-1}} \tag{2}$$

exists and compute it.