MATH 262 QUIZ II

1) Solve
i) $y' + 2ty = \sin(t)$
ii) $y' = 1/\sqrt{y}, y(1) = 1$
iii) $y' = y(2 - y)$
iv) $y'' + 4y' = t$
v) $y'' + 4y = t$
vi) $y'' - 4y = 0$
vii) $y'' - 4y^2 = 0$ just do the first integral.

2) Are the functions $f(t) = t^2 + 5t, g(t) = t^2 - 5t$ linearly independent? Can they be solutions of a single second-order differential equation? If so, write one down.

3) Solve
i) $y'' - 2y' - 3y = 3te^{-t}$
ii) $t^2y'' - 2y = 3t^2 - 1,$ for $t > 0$, verify $y_1(t) = t^2,$ and $y_2(t) = 1/t$ solve the homogeneous equation and find a particular solution.
iii) $(1 - x)y'' + xy' - y = g(x), 0 < x < 1,\ y_1(x) = e^x,\ y_2(x) = x,$ same instructions.
iv) $y'' + x^2y = 0, y(0) = 1, y'(0) = 0$ by series about zero. Get the radius of convergence of your solution.

4) Find the first 5 Fourier coefficients of the function that agrees with $f(x) = x^2$ on the interval $[0, 2\pi]$.

5) Same as 4 for $f(x) = 1$ on $[0, 1]$ and zero elsewhere.

6) Solve the system $y_1' = \alpha - \beta y_1, y_2' = \beta y_1 - \gamma y_2$ if all the Greeks are positive, what is the behavior of the solutions as $t \to \infty$?

7) Solve the two following simple pdes:
i) $\frac{\partial u}{\partial x} = 0,\ u(0, y) = \phi(y)$ (BVP)
ii) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$ (Get general solution)