DIFFERENTIAL EQUATIONS EXAM I

This exam is due at classtime on 5 April.

Defining

- Zip $\phi_0(x) = 0$,
- Box $\phi_1(x) = \begin{cases} 8 & x \in [7/16, 9/16] \\ 0 & x \notin [7/16, 9/16], \end{cases}$
- Tent $\phi_2(x) = \begin{cases} 2x & x \in [0, 1/2] \\ 2 - 2x & x \in [1/2, 1], \end{cases}$
- Zig-zag $\phi_3(x) = \begin{cases} 3x & x \in [0, 1/3] \\ 3 - 6x & x \in [1/3, 2/3] \\ 3x - 3 & x \in [2/3, 1], \end{cases}$

solve the following initial/boundary value problems as we have been, using *Maple*, for $u = u(x,t)$, $x \in [0,1], t > 0$. Do various interesting values of k, l, plot the results well, and provide physical explanations of the results. You may work in groups of two, at largest.

- (1+1 Dirichlet Heat) $\begin{cases} u_{xx} = u_t \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = \phi_k, \end{cases}$
- (1+1 Neumann Heat) $\begin{cases} u_{xx} = u_t \\ u_x(0,t) = u_x(1,t) = 0 \\ u(x,0) = \phi_k, \end{cases}$
- (1+1 Wave) $\begin{cases} u_{xx} = u_{tt} \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = \phi_k(x) \\ u_t(x,0) = \phi_l(x). \end{cases}$

For $u = u(x,y)$, $x, y \in [0,1]$, solve

- (2 Dirichlet Laplace) $\begin{cases} u_{xx} + u_{yy} = 0 \\ u(x,0) = u(0,y) = u(x,1) = 0 \\ u(1,y) = \phi_k(y). \end{cases}$

If you need help, do not hesitate to email, phone, or see me during office hours with **specific** questions.

Best,

JJP