

MATH 126 EXAM II

1) Evaluate the following (determining convergence for the improper \int s).

$$\text{i) } \int_1^{\infty} \frac{dx}{x(\ln x)^2}, \quad \text{ii) } \int t^2 \sin(t) dt, \quad \text{iii) } \int_{-\infty}^0 te^t dt, \quad \text{iv) } \int_1^{\infty} \frac{dt}{t^2 - 1},$$

$$\text{v) } \int_1^{10} \frac{t dt}{\sqrt{t^2 - 1}}.$$

2) Determine whether the following converge:

$$\text{i) } \int_0^{\infty} t^2 \sin(t) dt, \quad \text{ii) } \int_0^{\infty} t^{-2} \sin(t) dt, \quad \text{iii) } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}, \quad \text{iv) } \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3 - 1}},$$

$$\text{v) } \sum_{n=1}^{\infty} \frac{1}{n(n+1)}, \quad \text{vi) } \sum_{n=1}^{\infty} \frac{3^n + 5^n}{6^n}, \quad \text{vii) } \sum_{n=1}^{\infty} \frac{n+1}{n^3 + 3}.$$

3) Geometry Problems:

i) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

ii) Find the volume of the ellipsoid obtained by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x -axis. How about the y -axis? *Hint:* $a \leftrightarrow b$.

iii) Find the volume of the solid swept out by dragging the ellipsoid from part (ii) along the x axis c units.

iv) How much work is required to pump all the water out of the ellipsoid if the units are in feet?

v) Find an integral expression for the perimeter (arc-length) of the ellipse in part (i). Do the same for the surface area of the solids in parts (ii) and (iii).

4) A tank contains 1000 l of pure water. A NaCl solution of concentration .05 kg/l enters the tank at a rate of 6 l/min . Assuming that the solution in the tank is mixed instantaneously and that the solution drains from the tank at 6 l/min , find the concentration of NaCl in the tank as a function of t .

5) Solve the following differential equations:

$$\text{i) } \frac{dx}{dt} = \frac{t}{x} e^t, \quad x(0) = 1, \quad \text{ii) } \frac{dy}{dt} = \frac{y \cos(x)}{1 + y^2}, \quad y(0) = 1, \quad \text{iii) } \frac{dy}{dt} = y(1 - y).$$

6) Find the interval of convergence for the following:

$$\text{i) } \sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}, \quad \text{ii) } \sum_{n=2}^{\infty} \frac{x^n}{\sqrt{n^3-1}}, \quad \text{iii) } \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}, \quad \text{iv) } \sum_{n=1}^{\infty} \frac{3^n + 5^n}{6^n} x^n.$$