

## MATH 125 FINAL EXAM

Tuesday 18 Dec 2001, 11:30 a.m. - 1:15 p.m. (#)  
SHOW ALL WORK CLEARLY ON ANS SHEET

1) Find the critical points of the following functions.

i)  $f_1(x) = \sqrt{x} + \sqrt{4-x}$

ii)  $f_2(x) = \frac{x}{x+1}$

iii)  $f_3(x) = e^{\ln x^2}$

iv)  $f_4(x) = -\ln(x^2 + 1)$

v)  $f_5(x) = e^{x^2+1}$

vi)  $f_6(x) = x \ln(x)$

vii)  $f_7(x) = \ln \sqrt{\frac{x}{2x+3}}$

2) Which of the above functions is/are invertible? Invert those for which it is possible. Which of the above functions is/are continuous everywhere? What are the points of discontinuity of those that are not?

3) Find the equation of the line tangent to  $f(x) = \ln(x+1)$ , at  $(0,0)$ . Evaluate  $\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x}$ . Do these jibe?

4) Find the equation of the line tangent to  $f(x) = \frac{(2x^2-1)\ln(x)}{x}$  at the point  $(1,0)$ .

5) Determine the limits

i)  $\lim_{x \rightarrow 0^+} f_1(x)$

ii)  $\lim_{x \rightarrow -1^+} f_2(x)$

iii)  $\lim_{t \rightarrow 0} \frac{f_3(x+t) - f_3(x)}{t}$

iv)  $\lim_{x \rightarrow 0} f_6(x)$

v)  $\lim_{x \rightarrow \infty} f_7(x)$

vi)  $\lim_{x \rightarrow 0} \frac{\sin^2 px}{\sin qx}$

vii)  $\lim_{x \rightarrow 0} \frac{1 - \cos^2 px}{\sin^3 qx}$

viii)  $\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))^2}{\ln(x)}$

ix)  $\lim_{x \rightarrow \infty} \sqrt[x]{x^p}$

x)  $\lim_{x \rightarrow -\infty} xe^x$

6) Find  $y'$  in the following.

i)  $y \cosh(y/a) = x/a$ ,

ii)  $\sin(x-y) = xe^x$

| Function                          | ****Critical Points****<br>if any | $f_k^{-1}$<br>if it exists | Points of Discont.<br>if any |
|-----------------------------------|-----------------------------------|----------------------------|------------------------------|
| $f_1 = \sqrt{x} + \sqrt{4-x}$     |                                   |                            |                              |
| $f_2 = \frac{x}{x+1}$             |                                   |                            |                              |
| $f_3 = e^{\ln x^2}$               |                                   |                            |                              |
| $f_4 = -\ln(x^2 + 1)$             |                                   |                            |                              |
| $f_5 = e^{x^2+1}$                 |                                   |                            |                              |
| $f_6 = x \ln(x)$                  |                                   |                            |                              |
| $f_7 = \ln \sqrt{\frac{x}{2x+3}}$ |                                   |                            |                              |

7) Find the dimensions of the rectangle having area equal  $1000\text{m}^2$  with the smallest possible perimeter.

| Problem       | *****Correct Solution***** |
|---------------|----------------------------|
| 3             | $l$<br>lim                 |
| 4             |                            |
| 5 <i>i</i>    |                            |
| 5 <i>ii</i>   |                            |
| 5 <i>iii</i>  |                            |
| 5 <i>iv</i>   |                            |
| 5 <i>v</i>    |                            |
| 5 <i>vi</i>   |                            |
| 5 <i>vii</i>  |                            |
| 5 <i>viii</i> |                            |
| 5 <i>ix</i>   |                            |
| 5 <i>x</i>    |                            |
| 6 <i>i</i>    |                            |
| 6 <i>ii</i>   |                            |
| 7             |                            |