Abstract

Let G be a unimodular Lie group, X a compact manifold with boundary, and M be the total space of a principal bundle $G \to M \to X$ so that M is also a strongly pseudoconvex complex manifold. In this work, we show that if G acts by holomorphic transformations in M, then the Laplacian $\square = \overline{\partial}^* \overline{\partial} + \overline{\partial}\overline{\partial}^*$ on M has the following properties: The kernel of $\square$ restricted to the forms $\Lambda^{p,q}$ with $q > 0$ is a closed, G-invariant subspace in $L^2(M, \Lambda^{p,q})$ of finite G-dimension. Secondly, we show that if $q > 0$, then the image of $\square$ contains a closed, G-invariant subspace of finite codimension in $L^2(M, \Lambda^{p,q})$. These two properties taken together amount to saying that $\square$ is a G-Fredholm operator. It is a corollary of the first property mentioned that the reduced L^2-Dolbeault cohomology spaces $L^2\bar{H}^{p,q}(M)$ of M are finite G-dimensional for $q > 0$.